Spacetime and Gravity: Assignment 7 Solutions

November 8, 2013

1.

Just insert $d\bar{t}$ into the Eddington metric and show it then gives Schwarzchild- or vice versa.

$$d\bar{t} = dt + \frac{2Gm}{r - 2Gm}dr \tag{1}$$

$$d\bar{t}^2 = dt^2 + (\frac{2Gm}{r - 2Gm})^2 dr^2 + \frac{2Gm}{r - 2Gm}dtdr \tag{2}$$

Then,

$$-(1 - \frac{2Gm}{r})d\bar{t}^2 + \frac{4Gm}{r}d\bar{t}dr + (1 + \frac{2Gm}{r})dr^2 \tag{3}$$

$$= -(1 - \frac{2Gm}{r})dt^2 + (1 - \frac{2Gm}{r})^{-1}dr^2 \tag{4}$$

2.

For an equation of state $p = w\rho$, we then can use this to relate the two FRW equations, which implies:

$$-w3\dot{R}^2 = 2(\ddot{R}R + \dot{R}^2) \tag{5}$$

Ansatz for R(t):

$$R = R_0 t^n \tag{6}$$

then

$$\dot{R} = t^{-1}nR \qquad \ddot{R} = t^{-2}n(n-1)R \tag{7}$$

Insert this into the equation above gives an equation relating n and w.

$$n = 2/3\frac{1}{1+w} \tag{8}$$

Or

$$w = \frac{2}{3}n^{-1} - 1 \tag{9}$$

For n=1/6, w= 3.