

PHY308: Space, Time, and Gravity - Week 2 Homework

In what follows, unless otherwise stated, we will use units such that the speed of light, $c=1$.

Problem 1 (15 marks)

In Kings I 7:23, in describing Solomon's temple there is a description of a circular "sea of cast bronze". It is described as follows: *It was round in shape, the diameter from rim to rim being ten cubits and it took a line thirty cubits long to go around it.*

Assume the bible is correct and explain the seemingly incorrect value of the ratio of the circumference to the diameter of a circle by postulating that it was built on the surface of a sphere of radius a .

Find the radius of this sphere a , in cubits to two decimal place. You may use:

$$c = 2\pi a \sin\left(\frac{r}{a}\right). \quad (1)$$

Hint: expand the above formula for $r < a$.

Problem 2 (10 marks)

Define,

$$z = x + \tau y \quad (2)$$

with τ a complex number given by, $\tau = \tau_1 + i\tau_2$ where τ_1 and τ_2 are real. What is the line element:

$$ds^2 = dz \bar{dz} \quad (3)$$

in terms of dx, dy, τ_1, τ_2 (where $\bar{dz}$ is the complex conjugate of dz .)

Problem 3 (25 marks)

- a. Carry out the following coordinate transformations on the flat space line element,

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2, \quad (4)$$

$$x = r \sin(\theta) \cos(\phi) \quad y = r \sin(\theta) \sin(\phi) \quad z = r \cos(\theta), \quad (5)$$

to get the line element in terms of the coordinates t, r, θ, ϕ . These are 3d spherical polars. [15 marks]

- b. Now, do another coordinate transformation to express the metric in terms of u, v, θ, ϕ where

$$u = t - r \quad v = t + r, \quad (6)$$

u, v are called light cone coordinates. Why? [10 marks]