Queen Mary University of London

**BSc/MSci Examination by Course Unit**

**Thursday $20^{\mathrm{th}}$ May 2013 14:30 - 17:00**

**PHY4215 Quantum Physics Duration: 2 hours 30 minutes**

**YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.**

## Instructions:

**Answer ALL questions from Section A. Answer ONLY TWO questions from Section B. Section A carries 50 marks, each question in section B carries 25 marks.**

**If you answer more questions than specified, only the first answers (up to the specified number) will be marked. Cross out any answers that you do not wish to be marked.**

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

**Important note:** The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from QMUL.

Please check now to ensure you do not have any notes, mobile phones or unauthorised electronic devices on your person. If you have any, raise your hand and give them to an invigilator immediately. It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption is also an assessment offence.

**EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM.**

**Examiners:**
Prof G Travaglini and Dr B Wecht

**SECTION A** **Answer ALL questions in Section A**

### Question A1

What is the energy, expressed in eV, of a photon of wavelength $\lambda = 1\,$nm?

**[5 marks]**

### Question A2

State Wien's displacement law for a blackbody.

**[5 marks]**

### Question A3

What was Planck's revolutionary hypothesis for the energy of the oscillators associated with the radiation emitted by a blackbody?

**[5 marks]**

### Question A4

A monochromatic beam of X-rays of wavelength $\lambda = 5\,$nm hits an aluminium plate. Given that the work function for aluminium is $W = 4.08$ eV, will electrons be emitted due to the photoelectric effect? Support your conclusion with a calculation.

**[5 marks]**

### Question A5

Find the de Broglie wavelength of an electron whose total energy is equal to 10 GeV. The mass of the electron is $m \simeq$ 0.5 MeV/$c^2$. Can you use a non-relativistic approximation to describe the electron?

**[5 marks]**

### Question A6

Consider an idealised single-slit experiment performed with electrons of de Broglie wavelength $\lambda$. The width of the slit is $a$. What condition should $\lambda$ obey in order to exhibit diffraction?

**[5 marks]**

### Question A7

Write down the time-dependent Schrödinger equation for the wavefunction $\psi(x,t)$ of a particle of mass $m$ moving in one dimension $x$ in the potential $V(x)$.

**[5 marks]**

**Question A8**

Write down the normalisation condition for the wavefunction $\psi(x)$ of a particle which is restricted to move in the one-dimensional interval $-a \leq x \leq a$.

**[5 marks]**

**Question A9**

A particle is constrained to move in the one-dimensional interval $-a \leq x \leq a$. Write down the definition of the expectation value $\langle x \rangle_\psi$ of the coordinate $x$ when the wavefunction of the particle is $\psi(x)$.

**[5 marks]**

**Question A10**

The wavefunction $\psi(x) = A\,x^2\,e^{-x}$ describes a particle which propagates in one dimension and is confined to the region $x > 0$. Find the most probable value of $x$.

**[5 marks]**

**SECTION B** **Answer TWO questions from Section B**

## Question B1

**(i)** Consider Bohr's model of the hydrogen atom. Assume that the nucleus is at rest. By identifying centripetal and electrostatic effects, find the classical relation between the distance $r$ of the electron from the nucleus and the electron speed $v$.

**[5 marks]**

**(ii)** Use the result derived in part (i) to eliminate the velocity $v$ from the expressions for the classical energy and angular momentum of the electron, thus giving these quantities as functions of the distance $r$ of the electron from the nucleus.

**[6 marks]**

**(iii)** Use Bohr's quantisation condition for the angular momentum to find an expression for the radius $r_n$ and the velocity $v_n$ of the electron in the $n^{\mathrm{th}}$ Bohr orbit.

**[7 marks]**

**(iv)** Consider now a doubly ionised Lithium atom $\mathrm{Li}^{++}$, that is, an ion composed of a nucleus of charge $3|e|$ and a single electron. Here, $e$ denotes the electron charge. Calculate the energy of the ground state expressed in eV. Write down the ratio of the radius of the $n^{\mathrm{th}}$ Bohr orbit for the ionised Lithium atom to the radius of the $n^{\mathrm{th}}$ Bohr orbit for the hydrogen atom.

**[7 marks]**

## Question B2

Consider the Compton scattering of a beam of X-ray photons of wavelength $\lambda = 0.03$ nm on a free electron which is initially at rest.

**(i)** Write down the relation between the energy $E$ of the initial photon and its wavelength $\lambda$, and find the energy of the X-rays expressed in eV. Write down the energy conservation equation for the process and use it to argue that the wavelength of the final photons is larger than the wavelength of the initial photons.

**[6 marks]**

**(ii)** Determine the shift in the wavelength of the observed radiation when the photons are scattered through an angle $\theta = 90^\circ$. Find also the kinetic energy of the recoiling electron. You may find the following formula useful:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) ,$$

where $\lambda'$ is the wavelength of the scattered radiation.

**[4 marks]**

**(iii)** Write down momentum conservation for the process. Use it to prove that the angle $\phi$ formed by the direction of the outgoing electron with the direction of the incoming photon satisfies the relation

$$\tan\phi = \frac{E'\sin\theta}{E - E'\cos\theta} .$$

Here $E$ and $E'$ are the energies of the incoming and outgoing photons, respectively.

**[7 marks]**

**(iv)** Determine the expression of the momentum component $p_y^e$ of the electron after the scattering as a function of the energy $E$ of the incident photon and the angle $\theta$. Here $\hat{y}$ is the direction orthogonal to that of the incident photon beam.

**[8 marks]**

**Question B3**

Consider a particle of mass $m$ which is confined to move freely in the one-dimensional interval $0 \leq x \leq L$. In other words, the potential $V$ is given by $V = 0$ for $0 \leq x \leq L$ and $V = \infty$ for $x > L$ and $x < 0$.

**(i)** Write down the (time-independent) Schrödinger equation and the boundary conditions on the wavefunction.

**[6 marks]**

**(ii)** Consider the wavefunction

$$\psi(x) = A\Big[\sin\left(\frac{2\pi x}{L}\right) \;+\; 4\,\sin\left(\frac{6\pi x}{L}\right)\Big]\,.$$

Write $\psi(x)$ in terms of the normalised solutions to the Schrödinger equation

$$\psi^{(n)}(x) \;=\; \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)\,, \qquad n = 1, 2, \ldots$$

and determine the normalisation constant $A$.

**[7 marks]**

**(iii)** Determine the average value of the energy in the state described by the wavefunction in part (ii). It is useful to remember that the value of the energy $E_n$ corresponding to $\psi^{(n)}$ is $E_n = \frac{\hbar^2\pi^2}{2mL^2}\,n^2$, with $n = 1, 2, \ldots$.

**[6 marks]**

**(iv)** At the time $t = 0$ the energy is measured and is found to be equal to $18\pi^2\hbar^2/(L^2 m)$. Write down the normalised wavefunction after the measurement has taken place. Find the average value of $x$ after the measurement has taken place. You may find the following indefinite integral useful:

$$\int\! dz\, z\sin^2 z \;=\; \frac{z^2}{4} - \frac{\cos(2z)}{8} - \frac{z\sin(2z)}{4} \;+\; \text{constant}\,.$$

**[6 marks]**

**Question B4**

Let $\psi(x) = a\,(x-b)\sqrt{x+1}$ be the wavefunction of a particle which is confined to move freely on a line parameterised by $x$ in the interval $-1 \leq x \leq 1$. In other words, the potential $V$ is given by $V = 0$ for $-1 \leq x \leq 1$, and $V = \infty$ for $x > 1$ and $x < -1$. Here $a$ and $b$ are constants.

**(i)** Determine $b$ such that $\psi(x)$ obeys the correct boundary conditions for the above potential.

**[5 marks]**

**(ii)** Normalise the wavefunction.

**[6 marks]**

**(iii)** Calculate the expectation value of $x$ and of the momentum $p$. Find also the most probable value of $x$.

**[7 marks]**

**(iv)** Does the particular wavefunction $\psi(x)$ given above describe a state of fixed energy that is a stationary wave, i.e. a solution to the time-independent Schrödinger equation? Justify your answer with an explicit calculation.

**[7 marks]**

---

**End of Paper - An Appendix of 1 page follows**

## Appendix

### Useful data

| | | |
|---|---|---|
| Speed of light in free space | $c$ | $= 3.00 \times 10^{8}\ \mathrm{m} \cdot \mathrm{s}^{-1}$ |
| Planck's constant | $h$ | $= 6.63 \times 10^{-34}\ \mathrm{J} \cdot s$ |
| Boltzmann's constant | $k$ | $= 1.38 \times 10^{-23}\ \mathrm{J} \cdot \mathrm{K}^{-1} = 8.63 \times 10^{-5}\ \mathrm{eV} \cdot \mathrm{K}^{-1}$ |
| Mass of the electron | $m$ | $= 0.5\,\mathrm{MeV}/c^2$ |
| | $\hbar c$ | $= 197\,\mathrm{MeV} \cdot \mathrm{fermi}$ |
| | $1\,\mathrm{eV}$ | $= 1.6\,10^{-19}\,\mathrm{J}$ |
| | $1\,\mathrm{MeV}$ | $= 10^{6}\,\mathrm{eV}$ |
| | $1\,\mathrm{fermi}$ | $= 10^{-15}\,\mathrm{m}$ |
| | $1\,\mathrm{nm}$ | $= 10^{-9}\,\mathrm{m}.$ |