Queen Mary
University of London

BSc/MSci EXAMINATION

PHY-215 QUANTUM PHYSICS

Time Allowed: 2 hours 30 minutes

Date: 20^{th} May, 2011

Time: 14:30 - 17:00

Instructions: **Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 50 marks, each question in section B carries 25 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question. Course work comprises 20% of the final mark.**

A formula sheet containing mathematical results that may be of help in various questions is provided at the end of the examination paper.

Numeric calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Important Note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from the college. Please check now to ensure that you do not have any notes in your possession. If you have any then please raise your hand and give them to an invigilator immediately. Exam papers cannot be removed from the exam room

You are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Examiners: Dr. Rodolfo Russo (MO), Dr. Gabriele Travaglini (DMO)

SECTION A. Attempt answers to all questions.

A 1 What is the temperature of a black-body emitting mostly electromagnetic waves of wavelength $\lambda \sim 1.06\ 10^{-3}$ m? [5]

A 2 Draw a sketch of the experimental apparatus used by Young to highlight the wave-like nature of light. How is the light wavelength related to measurable quantities in the experimental apparatus? [5]

A 3 The maximum wavelength of electromagnetic radiation that will result in the photoelectric emission of electrons from a sample of metal is 5×10^{-7}m. Find the maximum kinetic energy of the electrons if the incident radiation has a wavelength of 3×10^{-7}m. [5]

A 4 In Compton's experiment X-rays of wavelength λ interact with free electrons and after the scattering have wavelength $\lambda' > \lambda$. Derive the expression for the total momentum of the recoiling electron as a function of λ, λ' and the angle θ between the incident and the scattered X-rays. [5]

A 5 Electrons are accelerated through a potential V and then they are suddenly stopped and emit photons. Derive the minimal value that the potential should have in order to produce some X-rays photons of wavelength 10^{-10}m. [5]

A 6 In Bohr's model for the hydrogen atom, the energy levels of the electron are given by

$$E_n = -\frac{C}{n^2}\ , \text{ with } n = 1, 2, \dots\ ,$$

where $C \approx 13.6$ eV. Derive the frequency of the electromagnetic radiation emitted when the electron makes a transition from the third ($n = 3$) to second ($n = 2$) level. [5]

A 7 The de Broglie wavelength of a beam of electrons is $\lambda = 10^{-10}$ m. Calculate their kinetic energy. Justify any approximation you make. [5]

A 8 Use de Broglie's idea of standing waves to find the energy levels of a particle of mass m that is constrained to be in a one-dimensional box of size L. [5]

A 9 The wavefunction $\psi(x) = Axe^{-x}$ describes a particle which propagates in one dimension and is confined to the region $x > 0$. What is the most probable position for this particle? [5]

A 10 Use Heisenberg's uncertainty principle to estimate the average kinetic energy of an electron confined in a (one-dimensional) box of size $\approx 10^{-10}$ m. [5]

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SECTION B. Answer two of the four questions in this section.

B1

(a) State the Stefan-Boltzmann law for the total power per unit area $I(T)$ of the electromagnetic radiation emitted by a black-body at temperature T. [5]

(b) Planck's law gives the frequency (f) distribution of the black-body radiation, that is how each wavelength (λ) contributes to the intensity of the radiation emitted

$$I(f,T) = \frac{2\pi h}{c^2}\frac{f^3}{e^{\frac{hf}{k_BT}}-1} ,$$

where k_B is the Boltzmann constant. Use this result to derive the temperature dependence of $I(T)$ in Stefan-Boltzmann law (you are not required to derive the value of the Stefan-Boltzmann constant σ). [6]

(c) If N_1^{ph} and N_2^{ph} are the number of photons emitted by the same black-body at the temperature T and $2T$ respectively, calculate the ratio $N_1^{\text{ph}}/N_2^{\text{ph}}$. [6]

(d) If f_{max} is the peak of the intensity per unit frequency and λ_{max} is the peak of the intensity per unit wavelength, show that $f_{\text{max}} \neq c/\lambda_{\text{max}}$. [8]

B2

(a) Consider a particle of mass m that moves freely on a circle of radius R. Use x with $0 < x < 2\pi R$ to parameterize the circle and write down the (time-independent) Schrödinger equation for this particle and the boundary conditions for the wavefunction $\psi(x)$. [6]

(b) Find the general solution to the equation in part (a) and show that it implies the existence of an infinite number of discrete energy levels. [8]

(c) Consider now two non-interacting identical particles on the same circle. What is the ground state energy for this system if the two particles are bosons ? [5]

(d) What is the ground state energy $E_{g.f}$ if the particles in part (c) are subject to Pauli's exclusion principle? Write down the most general wavefunction for a state with energy $E_{g.f}$ (for this question, you do not need to fix the normalization of the wavefunction). [6]

B3

Consider a particle of mass m that is constrained to be in a one-dimensional box of size L, but is otherwise free to move inside the box. We parameterize the box with the coordinate x, with $0 < x < L$. The state of the particle is described by the wavefunction

$$\psi(x) = A\left[\cos\left(2\pi\frac{x}{L}\right) - 1\right] .$$

(a) Normalize ψ appropriately. [5]

(b) Does this wavefunction describe a state of fixed energy that is a stationary wave? Justify your answer. [6]

(c) What is the probability of finding the particle in the first quarter of the box? [6]

(d) Calculate the average kinetic energy of this particle. [8]

B4

Rydberg's formula $\frac{1}{\lambda_n} = R\left(1 - \frac{1}{n^2}\right)$, where R is the Rydberg constant, gives the wavelength λ_n of the electromagnetic radiation emitted by a hydrogen atom as its electron jumps from the n^{th} level to the first level (the ground state).

(a) The line corresponding to the transition from the second to the first level has a wavelength of 1.2×10^{-7}m. Calculate the ionization energy of the hydrogen atom. [5]

(b) Consider Bohr's model of the hydrogen atom and write the electrostatic force on the electron and its classical equation of motion. By using the quantization of the angular momentum, find an expression for the energy of the n^{th} Bohr orbit. [9]

(c) Using the result obtained in (b), derive an explicit expression for the Rydberg constant in terms of the electron mass and charge. [5]

(d) Consider an ionized Helium atom He^+, that is an ion with a nucleus of charge $2|e|$ and a single electron. What would the Rydberg constant be for the electromagnetic radiation emitted by this ion? [6]

FORMULA SHEET

Speed of light in free space	$c = 3.00 \times 10^8$ m·s^{-1}
Wien's constant	$\approx 2.9 \times 10^{-3}$ m·K
Planck's constant	$h = 4.14 \times 10^{-15}$ eV·s
	$\hbar = 6.59 \times 10^{-16}$ eV s
Electron rest mass	$m_e \approx 0.5$ MeV$/c^2$
electric constant	$\epsilon_0 \approx 137 e^2/(2hc)$
Electron charge	$e = -1.6 \times 10^{-19}$ C
	1 eV $= 1.6 \times 10^{-19}$ J

End of paper