Quantum Physics PHY4215- Exercise Sheet 9

1. Consider an electron trapped in a one-dimensional region of length 1.00×10^{-10}m $=$ 0.100 nm. (a) In the ground state, what is the probability of finding the electron in the region from $x = 0.0090$ nm to 0.0110 nm ? (b) In the first excited state, what is the probability of finding the electron between $x = 0$ and $x = 0.025$ nm ?

2. For the eigenstate

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

show that the expectation values $< x >, < x^2 >$ are

$$< x > = \frac{L}{2}$$
$$< x^2 > = L^2(\frac{1}{3} - \frac{1}{2\pi^2 n^2})$$

Hints : For calculating $< x >$ use integration by parts. Calculating $< x^2 >$ can also be done using integrating by parts. You may use the formula

$$\int_0^L x^2 \sin^2(\frac{n\pi x}{L}) dx = L^3(\frac{1}{6} - \frac{1}{4\pi^2 n^2})$$

3. A particle in a box is in a quantum state which is a superposition of two states ψ_n, ψ_m where $n \neq m$

$$\psi = c_1\psi_n + c_2\psi_m$$

where c_1, c_2 are two complex numbers. What is the probability of measuring the energy to E_n and E_m respectively ? (This follows from one of the postulates of Quantum physics discussed in class).

If we make a large number of measurements N, estimate the number of times the measurement will give E_m and the number of times it will give E_n. What is the average energy measured by averaging over all the N measurements ?

This is the same thing as the expectation value of the energy. Show that the expectation value obtained above is consistent with the rule

$$< \hat{E} > = \frac{\int \psi^* (\hat{E}\psi) dx}{\int \psi^* \psi dx}$$

4. A general solution of the equation

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

can be written as a linear combination (superposition) of two distinct solutions, e.g $\sin kx, \cos kx$

$$\psi = A \sin kx + B \cos kx$$

Alternatively, we can use the pair of solutions e^{ikx}, e^{-ikx}. So the same ψ can be written as a superposition

$$\psi = A' e^{ikx} + B' e^{-ikx}$$

Express A', B' in terms of A, B. Use the equation

$$e^{\pm i\theta} = \cos\theta \pm i \sin\theta$$

or equivalently

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$
$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$