QUANTUM MECHANICS A (SPA-5319)

Field Emission

Let us consider the case where the surface of a metal with work function φ_1 experiences an electric field of strength ε. The electrons at the Fermi level of the metal do not have sufficient energy to overcome the potential barrier and be thermally liberated (the back of the envelope calculation, taking φ_1 to be of the order of 1eV and room temperature, $k_B T \sim$ 25meV, gives a probability of $e^{-40} \sim 10^{-18}$).

Such a situation is shown in figure 1. We notice, however, that the electrons are able to tunnel through the potential barrier defined by the work function and the vacuum level.

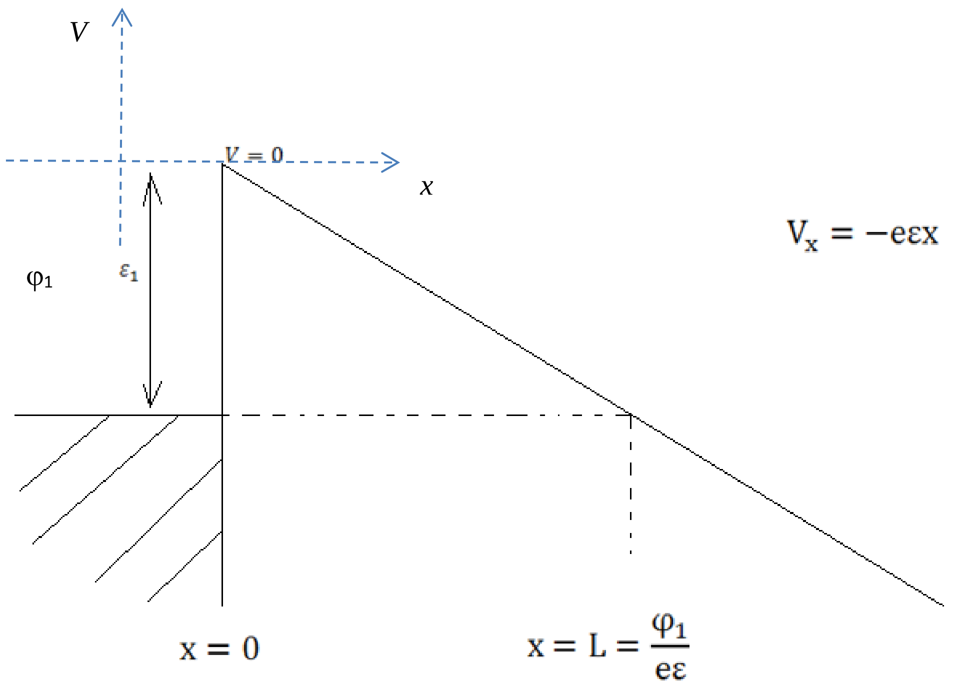

Figure 1: *Electrons from the Fermi level of a metal can tunnel through to the vacuum level if the electric field strength, ε, is sufficiently large.*

Placing the origin of our coordinate system at the metal surface, and taking the vacuum level to have zero potential, allows us to calculate the Gamow factor for tunneling in such a situation. The classically forbidden region extends from zero to a tunnelling distance L, where the electrons emerge and are subsequently accelerated by the electric field.

The Gamow factor thus becomes: $G = 2\sqrt{\frac{2m}{\hbar^2}}\int_0^L \sqrt{\varphi_1 - e\varepsilon x}\, dx = 2\sqrt{\frac{2m}{\hbar^2}}\int_0^L \varphi_1^{\frac{1}{2}}\sqrt{1 - \frac{e\varepsilon x}{\varepsilon_1}}\, dx,$

where we have calculated the tunnelling distance by equating the electron energy to the potential $L = b = \frac{e\varepsilon x}{\varphi_1}$. This yields the result $G = \frac{4}{3}\sqrt{\frac{2m}{\hbar^2}} \cdot \frac{\varphi_1^{\frac{3}{2}}}{e\varepsilon}$.

Consider now a realistic situation, where a large voltage bias, V_A , is applied between a tip and a screen at a distance D, as shown in figure 2.

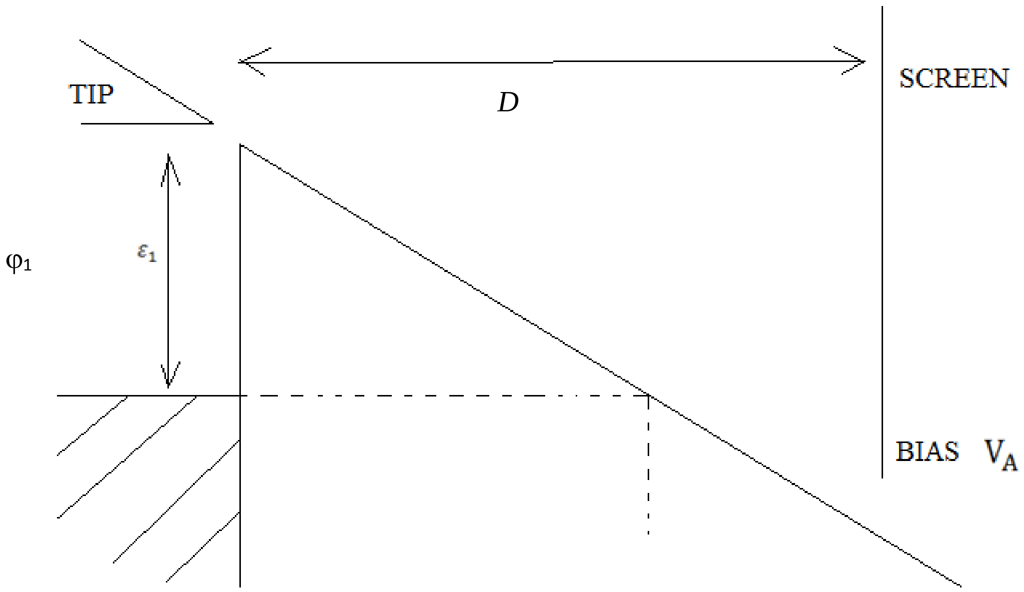

Figure 2: *A realistic field emission set-up.*

The maximum electric field strength is given by $\varepsilon = \frac{\mathrm{V_A}}{\mathrm{D}}$, and this allows us to write the Gamow factor in terms of the bias as $\mathrm{G} = \frac{4}{3}\sqrt{\frac{2\mathrm{m}}{\hbar^2}} \cdot \frac{\varepsilon_1{}^{\frac{3}{2}}}{\mathrm{e}} \cdot \frac{\mathrm{D}}{\mathrm{V_A}}$, and hence we have $\mathrm{G} \propto \frac{1}{\mathrm{V_A}}$. As the tunnelling probability $\propto e^{-G}$ we also have that the tunnelling current is proportional to e^{-G}, or $e^{-\frac{1}{V}}$, as shown in figure 3.

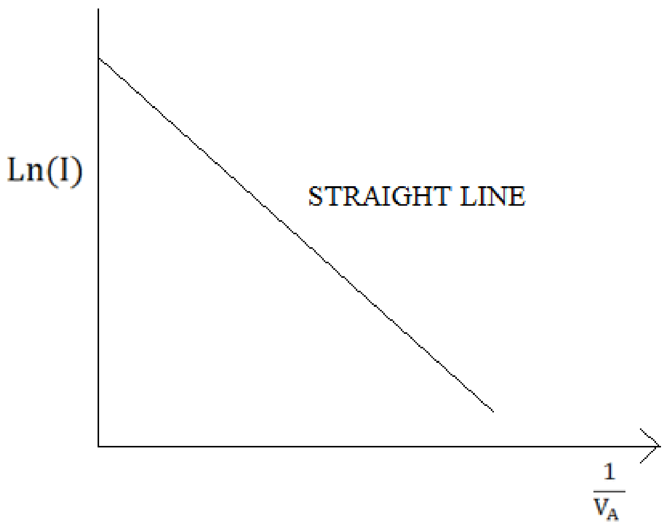

Figure 3: *The dependence of the field emission current I on the applied bias V_A.*