QUANTUM MECHANICS A (SPA 5319)

The Postulates of Quantum Mechanics

A postulate is a hypothesis. If it agrees with experiment, then it can be taken as an axiom (a truth that we cannot prove). Quantum Mechanics is based upon a number of postulates; these are stated quite differently depending on the source, and can vary in number depending on the book that one reads. In these notes I have used the following four postulates:

1. Every system can be described by a wavefunction (state function), $\Psi(r,t)$ ($\Psi(x,t)$ in 1D), which contains all accessible information about the system, including its spatial and temporal evolution.

2. The probability of finding a particle in a volume element dV, at position r and time t is

$$PdV = |\Psi(r,t)|^2 dV = \Psi^*(r,t)\Psi(r,t)dV \tag{1}$$

This means that $\Psi^*(r,t)\Psi(r,t)$ is a probability density function for our system ($\Psi^*(x,t)\Psi(x,t)$ in 1D). Since the particle must exist somewhere (anywhere), we obtain the normalisation condition: $\int_{-\infty}^{\infty} \Psi^*(r,t)\Psi(r,t)dV = 1$ (or $\int_{-\infty}^{\infty} \Psi^*(r,t)\Psi(r,t)dx = 1$, in 1D).

3. Every observable, q, is represented by an operator, $\hat{q}$, and this operator is used to obtain information about the observable. For example, the momentum operator is $\hat{p} = -i\hbar\partial/\partial x$, and the position operator is $\hat{x} = x$. The expectation value (or average) of any given observable is

$$\langle q \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\hat{q}\Psi(x,t)dx \tag{2}$$

and the uncertainty is given by $\Delta q = \sqrt{\langle q^2 \rangle - \langle q \rangle^2}$.

4. The wavefunction of an isolated system develops in time according to the time-dependent Schrödinger equation (TDSE):

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r,t) + V(r,t)\Psi(r,t) = i\hbar\frac{\partial\psi(r,t)}{\partial t} \tag{3}$$

which in 1D is given by $-\frac{\hbar^2}{2m}\frac{d^2\psi(x,t)}{dx^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$. This can sometimes appear in a more compact form, using the Hamiltonian (energy) operator $\hat{H}\Psi = i\hbar\partial\Psi/\partial t$.

As the wavefunctions from postulate 1 must obey the TDSE from postulate 4, there are important conditions that apply to $\Psi(x,t)$. These include, amongst others, that both $\Psi(x,t)$ and its derivative, $\partial\Psi(x,t)/\partial t$, must be continuous.